

Broadcasting Quantum Fisher Information

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It is well known that classical information can be cloned, but non-orthogonal quantum states cannot be cloned, and non-commuting quantum states cannot be broadcast. We conceive a scenario in which the object we want to broadcast is the statistical distinguishability, as quantified by quantum Fisher information, about a signal parameter encoded in quantum states. We show that quantum Fisher information cannot be cloned, whilst it might be broadcast even when the input states are non-commuting. This situation interpolates between cloning of classical information and no-broadcasting of quantum information, and indicates a hybrid way of information broadcasting which is of particular significance from both practical and theoretical perspectives.

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One of the most fundamental information tasks in communication is the dissemination of resources, such as quantum states [1–7], or correlations [8–12]. In contrast to the classical regime, quantum mechanics usually imposes strong limitations on such a task. Celebrated examples are no-cloning of non-orthogonal quantum states [1], no-broadcasting of non-commuting quantum states [2], and no-local-broadcasting of quantum correlations [10, 12]. However, in many theoretical and practical issues, it is not the states, but rather the information about a signal parameter encoded in the quantum states, that is needed to be disseminated. The information carried by a physical parameter is usually synthesized by quantum Fisher information (QFI) [13–22], which is the minimum achievable statistical uncertainty in the estimation of the parameter, and plays a fundamental and crucial role in both quantum foundation and quantum practice such as quantum metrology [23]. This motivates us to study the cloning and broadcasting of QFI.

In this Letter, we show that QFI cannot be cloned, whilst it might be broadcast even when the underlying states encoding the parameter are non-commuting. In particular, we prove that QFI can be infinitely broadcast if and only if it is uniform, whose exact meaning will be given later. Moreover, we identify all states arising from broadcasting of QFI. As a hybrid object lying between purely classical information and fully quantum information, QFI is of unique significance in quantum information processing. Our results shed new insights into the nature of QFI.

Quantum Fisher information.—To estimate a physical parameter encoded in a family of quantum states $\{\rho_\theta\}_{\theta \in \Theta}$, one performs a measurement $M = \{M_j\}$, mathematically described by a positive-operator-valued measure, on the states and then forms an estimator $\hat{\theta}$ of the parameter in terms of the measurement outcomes. The measurement M induces a parametric

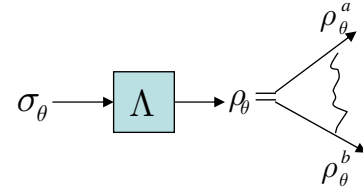


FIG. 1: The states σ_θ are “broadcast”, by the channel Λ , to the bipartite states ρ_θ , with reduced states $\rho_\theta^a = \text{tr}_b \rho_\theta$, $\rho_\theta^b = \text{tr}_a \rho_\theta$. While broadcasting states requires $\rho_\theta^a = \rho_\theta^b = \sigma_\theta, \forall \theta$, broadcasting QFI only requires $F(\rho_\theta^a) = F(\rho_\theta^b) = F(\sigma_\theta), \forall \theta$. Apparently, the former implies the latter, but the converse is not true in general.

probability distribution $p_\theta(j) := \text{tr} \rho_\theta M_j$ with the corresponding (classical) Fisher information $F(\rho_\theta|M) := \sum_j p_\theta(j) (\partial_\theta \log p_\theta(j))^2$. For an unbiased estimator $\hat{\theta}$, the celebrated Cramér-Rao inequality [13, 14], $\Delta \hat{\theta} := \langle (\hat{\theta} - \theta)^2 \rangle \geq 1/F(\rho_\theta|M)$, provides a fundamental limitation to the estimation precision. In order to achieve the best precision, one needs to maximize the Fisher information $F(\rho_\theta|M)$ over all measurements M . The maximum value is given by QFI [16], $F(\rho_\theta) := \text{tr} \rho_\theta L_\theta^2 = \max_M F(\rho_\theta|M)$, which is an intrinsic measure for the statistical distinguishability of states of the family $\{\rho_\theta\}$ in the neighborhood of ρ_θ . Here the symmetric logarithmic derivative (SLD) L_θ is determined by the equation $\partial_\theta \rho_\theta = (L_\theta \rho_\theta + \rho_\theta L_\theta)/2$. A measurement, which usually depends on the parameter, achieving the maximum Fisher information is called an optimal distinguishing measurement. Moreover, QFI is the infinitesimal metric of both the Bures distance $d_B(\rho_1, \rho_2) = \sqrt{2 - 2A(\rho_1, \rho_2)}$ [24], and the statistical distance [15–17]

$$d_S(\rho_1, \rho_2) := \max_M \arccos \sum_j \sqrt{\text{tr} \rho_1 M_j \cdot \text{tr} \rho_2 M_j} \quad (1)$$

with which a Riemannian geometry can be endowed on the space of states [25, 26]. Here $A(\rho_1, \rho_2) := \text{tr}(\rho_1^{1/2} \rho_2 \rho_1^{1/2})^{1/2} = \cos d_S(\rho_1, \rho_2)$ is the fidelity [27]. Consequently, for two neighboring states ρ_θ and $\rho_{\theta+d\theta}$, one has $d_B(\rho_\theta, \rho_{\theta+d\theta}) = d_S(\rho_\theta, \rho_{\theta+d\theta}) = \frac{1}{4} F(\rho_\theta) d\theta^2$. A measurement achieving the equality in Eq. (1) is called an optimal distinguishing measurement for ρ_1 and ρ_2 .

The task we consider is to disseminate the statistical distinguishability, as quantified by QFI, of a parameter $\theta \in \Theta$. The setup, as depicted in Fig. 1, is similar to the conventional cloning and broadcasting scenario: The states are “broadcast” to two parties a and b through a channel $\Lambda : \sigma_\theta \mapsto \rho_\theta \in S(H^a \otimes H^b)$ (state space). The reduced states for parties a and b are $\rho_\theta^a = \text{tr}_b \rho_\theta$, $\rho_\theta^b = \text{tr}_a \rho_\theta$, respectively. Unlike the conventional broadcasting, we are not interested in the quantum states themselves, but rather the physical parameter θ . We say that QFI of $\{\sigma_\theta\}_{\theta \in \Theta}$ can be cloned if there exists a channel $\Lambda : \sigma_\theta \mapsto \rho_\theta \in S(H^a \otimes H^b)$ such that (i) $F(\rho_\theta^a) = F(\rho_\theta^b) = F(\sigma_\theta)$; (ii) $\rho_\theta = \rho_\theta^a \otimes \rho_\theta^b$. We say that QFI can be broadcast if only (i) is required.

No-cloning of QFI.—First, we show that QFI cannot be cloned except for the trivial case of vanishing QFI. In fact, there exists a strong restriction to the distribution of QFI in case of factorized output states.

Theorem 1. *QFI cannot be cloned in the sense that for any channel $\Lambda : \sigma_\theta \mapsto \rho_\theta^a \otimes \rho_\theta^b$, if $F(\rho_\theta^a) = F(\sigma_\theta)$, then $F(\rho_\theta^b) = 0$. That is, if one party obtains the same QFI as the input states, then the other party cannot have any positive QFI.*

This is a consequence of the additivity of QFI for factorized states, $F(\rho_\theta^a \otimes \rho_\theta^b) = F(\rho_\theta^a) + F(\rho_\theta^b)$, and the monotonicity of QFI under quantum channels, $F(\rho_\theta^a \otimes \rho_\theta^b) \leq F(\sigma_\theta)$. Note that $F(\rho_\theta^b) = 0$ if and only if $L_\theta^b \sqrt{\rho_\theta^b} = 0$, which in turn implies that $\partial_\theta \rho_\theta^b = 0$. Here L_θ^b is SLD of ρ_θ^b . In this sense, we say that party b cannot have any information about the parameter. Theorem 1 may be viewed as a no-imprint principle for QFI. It is more general than the no-imprint principle for quantum states [4], where the restriction is keeping quantum states, stronger than keeping distinguishability.

Broadcasting of QFI.—In the broadcast scenario, the correlations between different parties are allowed. The broadcasting of quantum states always implies the broadcasting of QFI, but the converse is not true in general. In particular, in view of the no-broadcasting theorem for quantum states, namely, non-commuting states cannot be broadcast [2], QFI of a parametric family of commuting states can always be broadcast. On the other hand, since the previous no-broadcasting theorems show strong relevance with non-commutativity, one may ask whether the relation between non-commutativity and no-broadcasting is universal or not. Here, we show that QFI in some non-commuting quantum states can also be broadcast.

First, we consider an illustrative example. The family of equatorial states $\{\sigma_\theta = |\psi_\theta\rangle\langle\psi_\theta|\}_{\theta \in [0, 2\pi)}$ with $|\psi_\theta\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle)$ is apparently not a commuting family, and cannot be broadcast or cloned. Its approximate cloning has a remarkable application in quantum cryptography, through the link between cloning and eavesdropping [7, 28]. However, QFI of $\{\sigma_\theta\}$ can be broadcast: A Hadamard gate, followed by a controlled-NOT operation taking the input qubit as the source qubit and another qubit prepared in $|0\rangle$ as the target qubit, yields the states

$$|\Psi_\theta^{ab}\rangle = e^{i\frac{\theta}{2}} \left(\cos \frac{\theta}{2} |00\rangle - i \sin \frac{\theta}{2} |11\rangle \right),$$

from which we obtain $F(\sigma_\theta) = F(\rho_\theta^a) = F(\rho_\theta^b) = 1$, with $\rho_\theta = |\Psi_\theta^{ab}\rangle\langle\Psi_\theta^{ab}|$ and reduced states ρ_θ^a and ρ_θ^b .

It is of basic importance to determine for what kinds of parametric states, QFI therein can be broadcast. We first have the following observations. (i) QFI at a fixed parameter point, e.g., $\theta = \theta_0$, can be broadcast to any number of parties, through the broadcasting of the outcomes of an optimal distinguishing measurement $M_{\theta_0} = \{M_{j|\theta_0}\}$, which depends on θ_0 . In other words, via the channel

$$\sigma_\theta \mapsto \rho_\theta = \sum_j \text{tr} \sigma_\theta M_{j|\theta_0} \cdot (|j\rangle\langle j|)^{\otimes n} \quad (2)$$

we have $F(\rho_\theta^{(k)}) = F(\sigma_\theta)$ at $\theta = \theta_0$, where $\rho_\theta^{(k)}$ is the reduced states for party k . (ii) Unless the measurement M_{θ_0} in Eq. (2) is also optimal at other parameter points, the broadcasting channel (2) cannot be used to broadcast QFI at those parameter points. These observations indicate that the broadcasting of QFI is relevant to the dependence of the optimal distinguishing measurements on the parameter. This motivates us to introduce the following definitions: We say that QFI of $\{\sigma_\theta\}_{\theta \in \Theta}$ can be infinitely broadcast, if it can be broadcast to any number of parties. We say that QFI of $\{\sigma_\theta\}_{\theta \in \Theta}$ is uniform if there exists a single measurement, which is optimal for distinguishing any $\theta \in \Theta$, i.e., achieving the equality in $F(\rho_\theta) = \max_M F(\rho_\theta|M)$. Now our main result, which is reminiscent that a bipartite state is infinitely symmetric extendible if and only if it is separable [29, 30], may be stated as follows.

Theorem 2. *QFI of a parametric family of states $\{\sigma_\theta\}_{\theta \in \Theta}$ can be infinitely broadcast if and only if it is uniform.*

First, it is clear that if QFI is uniform, then it can be infinitely broadcast by the channel given by Eq. (2). To establish the converse, we assume without loss of generality that the parameter domain Θ is the unit interval $[0, 1]$. Consider the states $\{\sigma_{i/n}\}_{i=0,1,\dots,n}$, we first show that if the statistical distance between two neighboring states can be broadcast to m parties in the sense that there exists a channel $\Lambda : \sigma_{i/n} \mapsto \rho_{i/n} \in S(H^{\otimes m})$ such

that for all i and k ,

$$d_S(\sigma_{i/n}, \sigma_{(i+1)/n}) = d_S(\rho_{i/n}, \rho_{(i+1)/n}) = d_S(\rho_{i/n}^{(k)}, \rho_{(i+1)/n}^{(k)}),$$

then for every $m+1$ consecutive states in $\{\sigma_{i/n}\}_{i=0,1,\dots,n}$ there exists a single optimal measurement to distinguish the neighboring states. Here $\rho_{i/n}^{(k)}$ is the reduced state for party k . To prove this, let us first consider the case $m=2$ (two parties a and b). The broadcasting condition, $d_S(\rho_{i/n}, \rho_{(i+1)/n}) = d_S(\rho_{i/n}^{(k)}, \rho_{(i+1)/n}^{(k)})$, $k=a, b$, implies that there exist local optimal distinguishing measurements $M_{i,i+1}^a$ and $M_{i,i+1}^b$ for the two neighboring states $\rho_{i/n}$ and $\rho_{(i+1)/n}$. Moreover, $M_{i,i+1}^a \otimes M_{i+1,i+2}^b$ is an optimal distinguishing measurement for both the pairs of neighboring states $(\rho_{i/n}, \rho_{(i+1)/n})$ and $(\rho_{(i+1)/n}, \rho_{(i+2)/n})$, since the measurement-induced probability distribution for party a gives the largest possible distinction between states $\rho_{i/n}$ and $\rho_{(i+1)/n}$, while the one with respect to party b gives the largest possible distinguishability between $\rho_{(i+1)/n}$ and $\rho_{(i+2)/n}$, as illustrated in Fig. 2. In the meantime, note that if $M = \{M_j\}$ is an optimal distinguishing measurement for $\Lambda(\sigma_1)$ and $\Lambda(\sigma_2)$, then $\Lambda^\dagger(M) := \{\Lambda^\dagger(M_j)\}$ is an optimal distinguishing measurement for σ_1 and σ_2 . This is because performing $\Lambda^\dagger(M)$ on σ and performing M on $\Lambda(\sigma)$ yield the same probability distribution and thus the same distinguishability. Consequently, $\Lambda^\dagger(M_{i,i+1}^a \otimes M_{i+1,i+2}^b)$ is an optimal distinguishing measurement for both $(\sigma_{i/n}, \sigma_{(i+1)/n})$ and $(\sigma_{(i+1)/n}, \sigma_{(i+2)/n})$. This process can be straightforwardly extended to any m parties, and leads to the conclusion that if the statistical distance can be broadcast to m parties, then there exists a single optimal distinguishing measurement for every $m+1$ consecutive states. So the infinite-broadcasting of the statistical distance implies a single optimal distinguishing measurement for all states. When $n \rightarrow +\infty$, the statistical distance is proportional to QFI, and we obtain the desired result.

It should be emphasized that for finite broadcasting, the uniformness of QFI is not necessary for the broadcasting of QFI. In order to elucidate this, we first identify all states arising from broadcasting of QFI. We say that $\rho_\theta \in S(H^{\otimes n})$ is a family of n -partite QFI-broadcast states in the parameter domain Θ , if $F(\rho_\theta^{(k)}) = F(\rho_\theta)$ for every $\theta \in \Theta$ and every k . In what follows, we will take the notational convenience such as $X^a Y^{ab} = (X^a \otimes \mathbf{1}^b) Y^{ab}$.

Theorem 3. $\rho_\theta \in S(H^{\otimes n})$ is a family of n -partite QFI-broadcast states if and only if any SLD for the reduced states of any party is also an SLD for ρ_θ .

The sufficiency readily follows from the calculation of QFI. Indeed, if $L_\theta^{(k)}$, an SLD for $\rho_\theta^{(k)}$, is also an SLD for ρ_θ , then

$$F(\rho_\theta) = \text{tr } \rho_\theta L_\theta^{(k)2} = \text{tr } \rho_\theta^{(k)} L_\theta^{(k)2} = F(\rho_\theta^{(k)}).$$

To prove the necessity, we make use of the optimal distinguishing measurements. The condition $F(\rho_\theta) =$

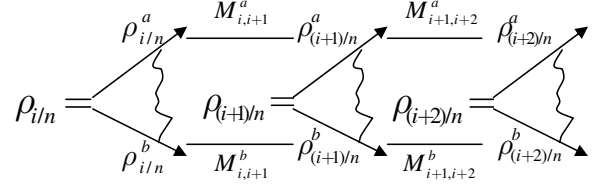


FIG. 2: Broadcasting of the statistical distance vs. the uniformness of QFI. $M_{i,i+1}^a$ and $M_{i,i+1}^b$ are local optimal distinguishing measurements for the local neighboring states $(\rho_{i/n}, \rho_{(i+1)/n})$ and $(\rho_{i/n}^b, \rho_{(i+1)/n}^b)$, respectively, $M_{i,i+1}^a \otimes M_{i+1,i+2}^b$ is an optimal global distinguishing measurement for both $(\rho_{i/n}, \rho_{(i+1)/n})$ and $(\rho_{(i+1)/n}, \rho_{(i+2)/n})$ due to the broadcasting condition. When n , which is the number of divisions for the parametric family states, tends to infinite, the statistical distance between neighboring states tends to QFI, and thus broadcasting infinitesimal statistical distance amounts to broadcasting QFI.

$F(\rho_\theta^{(k)})$ implies that there exists a local optimal measurement $M_\theta^{(k)} = \{M_{j|\theta}^{(k)}\}$ for distinguishing θ . On the other hand, it is known that $M_\theta = \{M_{j|\theta}\}$ is an optimal distinguishing measurement for θ if and only if [16]

$$M_{j|\theta}^{1/2} L_\theta \rho_\theta^{1/2} = u_{j|\theta} M_{j|\theta}^{1/2} \rho_\theta^{1/2}, \quad \forall j, \quad (3)$$

where $u_{j|\theta} := \partial_\theta \log \text{tr } \rho_\theta M_{j|\theta}$ when $\rho_\theta M_{j|\theta}$ is nonzero, and vanishes otherwise. Equation (3) implies that $L_\theta \rho_\theta = \left(\sum_j u_{j|\theta} M_{j|\theta} \right) \rho_\theta$, which in turn indicates that $\sum_j u_{j|\theta} M_{j|\theta}$ is an SLD for ρ_θ . Because $M_\theta^{(k)}$ is an optimal distinguishing measurement not only for $\rho_\theta^{(k)}$ but also for ρ_θ ,

$$L_\theta^{(k)} := \sum_j \partial_\theta \log \left(\text{tr } \rho_\theta M_{j|\theta}^{(k)} \right) \cdot M_{j|\theta}^{(k)} \quad (4)$$

is an SLD for both $\rho_\theta^{(k)}$ and ρ_θ . Moreover, every SLD for $\rho_\theta^{(k)}$ can be expressed in the form of Eq. (4), e.g., taking $M_{j|\theta}$ as the eigenstates of the SLD. Thus, for QFI-broadcast states ρ_θ , every SLD for $\rho_\theta^{(k)}$ is also an SLD for ρ_θ .

From Theorem 3, we can derive an important property of QFI-broadcast states: $[\rho_\theta^{(k)}, \partial_\theta \rho_\theta^{(k)}] = 0$ for the reduced states of any QFI-broadcast state ρ_θ .

To establish this, without loss of generality, we consider two parties a and b . According to Theorem 3, L_θ^a and L_θ^b are both SLD for ρ_θ . Due to the uniqueness of the product of SLD and the state [31], we have $L_\theta^a \rho_\theta = L_\theta^b \rho_\theta$, which implies $L_\theta^a \rho_\theta^a = \text{tr}_b L_\theta^a \rho_\theta = \text{tr}_b L_\theta^b \rho_\theta$. Similarly, we have $\rho_\theta^a L_\theta^a = \text{tr}_b \rho_\theta L_\theta^a = \text{tr}_b \rho_\theta L_\theta^b$. From the cyclic property of trace, $\text{tr}_b L_\theta^a \rho_\theta = \text{tr}_b \rho_\theta L_\theta^a$, we get $L_\theta^a \rho_\theta^a = \rho_\theta^a L_\theta^a$.

Now we have a better understanding of the relation between non-commutativity and no-broadcasting. QFI-broadcasting is necessary, but not sufficient, for the

broadcasting of quantum states. It is the requirement of the broadcasting of quantum states, i.e., $\rho_\theta^{(k)} = \sigma_\theta$, that transfers the commutativity from reduced states of the output states to the input states. This can also be seen from the original proof of the no-broadcasting theorem for quantum states [2].

The neighboring commutativity, $[\rho_\theta^{(k)}, \partial_\theta \rho_\theta^{(k)}] = 0$, most likely but not necessarily leads to the (pairwise) commutativity. If there exists a party for which the reduced states $\rho_\theta^{(k)}$ of the QFI-broadcast states are pairwise commuting, then the common eigenbases, with which all $\rho_\theta^{(k)}$ with $\theta \in \Theta$ are simultaneously diagonalized, make up a single local optimal distinguishing measurement for $\{\rho_\theta\}_{\theta \in \Theta}$. Thus, QFI of $\{\rho_\theta\}_{\theta \in \Theta}$ is uniform, which in turn implies the uniformity of QFI of input states $\{\sigma_\theta\}_{\theta \in \Theta}$, and hence their infinite broadcastability.

To show that the uniformness of QFI is not necessary for *finite* broadcasting of QFI, we explicitly construct a family of bipartite QFI-broadcast states whose QFI is not uniform. Without requiring the smoothness of the parametrization, such a family of two-qubit states can be constructed as $\sigma_\theta = |\Psi_\theta\rangle\langle\Psi_\theta|$ with

$$|\Psi_\theta\rangle = \begin{cases} |\psi_{yy}(-\theta)\rangle, & \text{if } \theta \in [-\pi/2, -\pi/4], \\ |\psi_{zz}(\theta)\rangle, & \text{if } \theta \in (-\pi/4, \pi/4], \\ |\psi_{xx}(\theta)\rangle, & \text{if } \theta \in (\pi/4, \pi/2], \end{cases}$$

where $|\psi_{ii}(\theta)\rangle = \cos\theta|i\rangle \otimes |i\rangle + \sin\theta|\bar{i}\rangle \otimes |\bar{i}\rangle$ for $i = x, y$ and z , $|x\rangle, |y\rangle$ and $|z\rangle$ are the eigenstates of the corresponding Pauli matrices with the eigenvalue 1, while $|\bar{x}\rangle, |\bar{y}\rangle$ and $|\bar{z}\rangle$ are those with eigenvalue -1 . QFI of $\{\sigma_\theta\}_{\theta \in [-\pi/2, \pi/2]}$ is not uniform, since there does not exist a single optimal distinguishing measurement for the whole domain, however, it can be broadcast by distributing the two qubits to two parties directly, since σ_θ are themselves bipartite QFI-broadcast states.

Discussion.—By measuring the statistical distinguishability of a parameter in terms of QFI, we have established a no-cloning theorem for QFI and a no-infinite-broadcasting theorem for non-uniform QFI. The results extend the no-broadcasting theorem for quantum states [2], and shed new lights on quantum communication and quantum metrology. It is also interesting to note that QFI-broadcasting is complementary to the environment-assisted precision measurement [32], whose essence is the concentration of QFI from the system-environment entirety into the system.

The broadcasting and manipulation of QFI is of broad significance for quantum information processing, opens new research directions, and indicates an alternative way in understanding the boundary and link between classical and quantum information.

Firstly, in practice, many important quantities are represented by parameters, which in turn are physically encoded in quantum states, and thus the estimation or measurement of the parameters, rather than the quantum

states themselves, are of practical relevance. It is much costly, and actually not necessary, to gain complete information about the quantum states. QFI information is a fundamental ingredient in such a scenario. For example, the phase estimation is an extremely important issue. It is the relative phase in superposition or coherence, rather than the superposed quantum states themselves that needs to be estimated or measured.

Secondly, in quantum foundations, a crucial and long standing problem is to depict the boundary between classical and quantum information, which actually lies in the heart of quantum decoherence and quantum measurement. This may first require our deeper understanding of quantities lying between classical and quantum nature. QFI is precisely such an intrinsic and hybrid character. In fact, QFI represents a kind of information intermediate between the classical parameters (which can be freely cloned and broadcast) and complete quantum information (represented by quantum states, but cannot be broadcast in general). Thus the understanding of QFI may serve as a bridge connecting our theories between classical and quantum and in particular may open a new avenue in attacking the manifestation of classicality from quantum substrate.

Thirdly, QFI broadcasting has immediate implications in some quantum metrology problems, where quantum sensors and measurement apparatus are connected by noise channels. In order to overcome the loss and increase the receiving efficiency, one can utilize the broadcasting technology to distribute information. However, broadcasting quantum states is not only very restricted, but also unnecessary in many cases. Broadcasting QFI relaxes the restrictions to a great extent, shows much more flexibility, and keeps the essence of information. More importantly, equatorial states, which are the most often used ones in quantum precision measurement, can be QFI-broadcast.

Finally, we mention the important subject of protection and transmission of quantum Fisher information in theoretical and experimental quantum parameter estimation, as opposed to the protection and transmission of quantum states, which are much more costly and complicated. For example, decoherence-free subspaces for phase estimation still focus on the protection of quantum states [33]. The situation will be quite different if we consider the protection of QFI instead. Our work can help to develop novel strategies for robust quantum enhanced phase estimation against noise.

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